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Perspective

Perspective: Applied Calculus for Public Health Epidemiology--Empowering Public Health Professionals with Mathematical Insights

Gregory V. Fant, PhD, MSHS*

US Department of Veterans Affairs, VHA/Patient Care Services, Population Health Services (10P4V), Washington, DC 20420, USA

***Corresponding author**
Gregory V. Fant, PhD, MSHS
Public Health Epidemiologist, US Department of Veterans Affairs, VHA/Patient Care Services, Population Health Services (10P4V), Washington, DC 20420, USA;
E-mail: gregory.fant2@va.gov

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ABSTRACT

This essay serves as a perspective and practical guide for public health epidemiologists, illustrating the application of essential calculus and statistical techniques to enhance analytical skills. It focuses on how derivatives, integrals, differential equations, and key statistical techniques can be directly applied to public health data analysis, improving decision-making and the effectiveness of health interventions. Derivatives are presented as tools for measuring the rate of change in epidemiological data, essential for tracking disease trends. Integrals are explained as means to aggregate data, offering a big-picture view to evaluate the overall effect of public health events over time. First-order differential equations are explored to understand the spread of significant public health diseases, supporting the development of epidemiologic models that inform public health responses. Furthermore, a combination of probability and calculus is shown to support a range of statistical methods, crucial for analyzing health data and interpreting outcomes. Aimed at demystifying these mathematical principles, this guide empowers all public health professionals to apply such knowledge in designing, evaluating, and refining public health interventions for the population. By harnessing these concepts, public health professionals can enhance their analytic and decision-making processes, significantly contributing to the advancement of community public health and well-being.

Keywords
Applied calculus; Public health epidemiology; Derivatives; Integrals; First-order differential equations; Statistical techniques.

INTRODUCTION

The landscape of public health epidemiology is evolving with the emergence of "Big Data" and advanced technology solutions,[1,2] including the advent of GenerativeAI.[3] Public health agencies, which amass vast data volumes to understand and address community health needs for the population, stand at the forefront of this transformation. As epidemiologists increasingly turn to technological tools for data management and analysis, it becomes pertinent to explore how applied calculus—encompassing derivatives, integrals, differential equations, probability, and statistics—can unlock additional insights from public health data.

This perspective aims to offer a collection of professional notes and insights, demonstrating how applied calculus can inform epidemiology in public health practice,[4] or public health epidemiology. By translating complex public health data into actionable insights, applied calculus serves as a vital resource for public health decision-makers. Through an examination and contextualization of content from standard texts on applied calculus,[5,6] this brief essay will discuss four critical topics: the derivative, integration, first-order differential equations, and the interplay of probability with applied calculus. Our goal is to provide epidemiologists and other public health professionals with an overview of applied calculus and an enhanced understanding of these mathematical concepts, bolstering their decision-making and intervention strategies in public health. In this perspective, we want to bring some ideas from applied calculus into the context of the professional workplace and explain the ideas as simply as possible. While not a substitute for formal calculus education, this work seeks to bridge applied calculus topics with prac-

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tical applications in epidemiology, facilitating a clearer connection to professional practice.

DIFFERENTIATION: THE DERIVATIVE

The Derivative in Public Health Epidemiology

Applied Calculus is a "*single-level course that covers the basics of several topics (from the study and application of calculus) such as functions, derivatives, and integrals*".[7] The derivative stands out as a critical tool in the health sciences, public health, and epidemiology for analyzing and interpreting nuanced changes in health-related data. It provides insights into the temporal dynamics of public health indicators, which are crucial for monitoring disease trends in populations and aiding in community diagnosis. In other words, the ability of the derivative to quantify the rate of change aids public health epidemiologists in constructing models for disease progression, forecasting health trends, and evaluating the impact of public health interventions in a population (Problem 1). Utilizing derivatives enables epidemiologists to discern points of acceleration or deceleration in the spread of communicable diseases, identify peaks in health-related events, and comprehend the rate at which public health metrics evolve.

Definition: Differentiation in the Context of Public Health Epidemiology

Formal Definition of Differentiation (Derivative): The derivative of a function f at a point x is defined as:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This definition captures the essence of differentiation as the limit of the average rate of change of the function as the interval approaches zero.

Differentiation, the process of finding a function's derivative, is pivotal in epidemiology. It allows for the examination of how a variable, such as the number of disease cases in a population, changes with respect to another variable, typically time. This insight is crucial for understanding the behavior of various public health phenomena, from the spread of communicable diseases in a population to the response of a population to a specific public health policy. By differentiating the cumulative number of disease cases over time given by a function, epidemiologists can determine the instantaneous rate at which new cases are occurring, providing a clearer picture of disease spread. This information can inform public health resource allocation, the identification, and development of public health intervention strategies, and help predict public health policy-making, ultimately aiding in disease control and public health prevention efforts in the population.

Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus links the concepts of differentiation and integration (discussed in the next section) and can be stated in two parts:

Part 1: If F is an antiderivative of f over an interval $[a,b]$, then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

Part 2: If f is continuous over $[a,b]$ and F is defined by

$$F(x) = \int_a^x f(t)dt$$

then F is differentiable on $[a,b]$, and $F'(x) = f(x)$

The Fundamental Theorem of Calculus bridges the concepts of differentiation and integration, showing that they are, in essence, inverse processes. It states that if a function is continuous over an interval and is the integral of its derivative on that interval, then the definite integral of the derivative is equal to the difference in the values of the original function at the bounds of the interval.

This theorem is significant because it provides a way to understand the cumulative effects of health policies, public health outcomes, or public health interventions over time. For the epidemiologist, the first derivative (the result of differentiating a function, f), is crucial for understanding the rate of change in public health data. There are two straightforward forms of the derivative:

Function Form, *f'(x)*: This notation shows the first derivative of a function f with respect to x. If f represents a public health metric over time, $f'(x)$ tells us how quickly that metric changes at any given time.

Differential Form, *dy/dx*: This form expresses the first derivative of y with respect to x, where y is a dependent variable, or outcome variable, representing the public health metric, and x is an independent variable, often time, t. The notation $\frac{dy}{dx}$ or dy/dx provides a general rate of change of y as x changes.

Either of these forms may be used to analyze trends, such as the rate of new infections in a population, enabling professionals to make informed decisions regarding disease control and public health prevention strategies.

Basic Rules of Differentiation

Understanding and applying the basic rules of differentiation is essential for epidemiologists analyzing variable changes over time, such as disease spread, vaccination rates, or other health-related events. The five fundamental rules are:

Constant rule: The derivative of a constant is zero.

$f(x) = c$ implies $f'(x) = 0$, where c is a constant

Imagine that a team of epidemiologists is considering a rare disease that is tracked or monitored by a public health surveillance system. The number of cases for this rare disease, *c*, is a very low number that does not change from month to month. When this rule of differentiation is applied, these professionals want to determine the number of new cases that change over time. Since the number of cases does not change from month to month, the rate of change *(f')* is zero.

Power rule: The derivative of *x* raised to a power is the power times *x* raised to one less than that power.

$$f(x)=x^n \text{ leads to } f'(x) = n \cdot x^{n-1}$$

Constant multiple rule: The derivative of a constant times a function is the constant times the derivative of the function.

$$\text{For } f(x)=c \cdot g(x),\ f'(x) = c \cdot g'(x), \text{ with c as a constant}$$

Sum rule: The derivative of the sum of two functions is the sum of their derivatives.

$$\text{If } f(x) = g(x) + h(x), \text{ then } f'(x) = g'(x) + h'(x)$$

Product rule: The derivative of the product of two functions is the first derivative of the first function times the second function, and the first function times the first derivative of the second function.

$$\text{Given } f(x) = g(x) \cdot h(x), \text{ we find } f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

These rules enable the modeling and understanding of complex changes within public health data. Understanding these rules enables epidemiologists to build models that describe and predict public health phenomena, which is crucial for effective public health interventions.

Problem 1: Using the derivative as a rate of change. Suppose that a flu outbreak impacts a town. This is a concern for the public health agency. The public health epidemiologist estimates that the number of persons sick with the flu at time t (measured in days from the beginning of the outbreak) is approximated by $P(t) = 60t^2 - t^3$, provided that $0 \leq t \leq 40$. At what rate is the flu spreading at time t = 30?

Solution:

1. Find the first derivative P'(t):

To find P'(t), we differentiate $P(t) = 60t^2 - t^3$ or $P(t) = 60t^2 \pm t^3$) with respect to t:

$$P'(t) = \frac{d}{dt}(60t^2) - \frac{d}{dt}(t^3) \qquad \text{(using the sum rule)}$$

Using the power rule for differentiation, then,

$$P'(t) = (2)(60t^{2-1}) - (3)(t^{3-1})$$

$$P'(t) = 120t - 3t^2$$

2. Evaluate P'(t) at t = 30:

Now, substitute t = 30 into P'(t) to find the rate at which the flue is spreading on Day 30:

$$P'(30) = (120)(30) - (3)(30^2)$$
$$P'(30) = 3600 - (3)(900)$$
$$P'(30) = 3600 - 2700$$
$$P'(30) = 900$$

So, at t=30 days, the flu is spreading in the town at a rate of 900 new cases per day.

Interpretation for public health epidemiology: Understanding how the incidence rate is changing is crucial for public health planning and response. If the incidence is increasing, more aggressive public health prevention measures might be needed. Conversely, if the incidence is decreasing, it might suggest that the outbreak is coming under control.

In this case, the fact that the epidemiologist has determined that 900 new cases are increasing per day at the 30-day mark might provide public health decision-makers with useful information to assess the outbreak's severity in the community and, accordingly, re-allocate public health resources. This problem highlights the importance of determining the rate at which an infectious disease spreads within a community using data from a public health surveillance system along with mathematical insights from applied calculus.

INTEGRATION: THE INTEGRAL

Integration: Aggregating Data Over Time

Integration offers a holistic view of disease impact over time, which is essential for assessing the effectiveness of public health interventions and strategies. It helps sum up changes, offering insights into the total effect of a public health outcome over a specified period.

Definition: Integration in the Context of Public Health Epidemiology

Integration is a fundamental concept that essentially represents the accumulation or summation of quantities over an interval. Formally, the integral of a function *f(x)* over an interval *[a,b]* is the limit of the sum of the areas of rectangles under the curve as the width of the rectangles approaches zero. If *F(x)* is an antiderivative of *f(x)*, the definite integral of *f(x)* from *a* to *b* is given by:

$$\int_a^b f(x)dx = F(b) - F(a)$$

This expression states that the definite integral of *f(x)* from *a* to *b* is equal to the difference in the values of its antiderivative *F* evaluated at *b* and *a*.

Integration is a mathematical method used to combine or accumulate quantities over a period. In other words, integral calculus helps to mathematically compile data, like the rate of new infections, over time to determine the total impact of a disease or the overall effectiveness of a health intervention. By integrating the rate of new infections over a specific period, epidemiologists can calculate the area under the curve, the total number of new cases during that time, offering essential insights for public health planning and response strategies (Problem 2).

Basic Rules of Integration

These rules form the foundation of integrating functions in calculus, aiding in the analysis and interpretation of epidemiological data.

Constant rule of integration: When we integrate a constant, *a* over an interval, the result is the constant multiplied by the variable of integration plus some arbitrary constant with the form

$$\int a\, dt = at + C$$

where the term *C* is known as the constant of integration. The constant, *a*, does not change. When we integrate the constant, we multiply the constant by the variable of integration. This rule helps to simplify the process of aggregating constant rates of change, a common activity in public health resource planning and epidemiological modeling. In public health epidemiology, imagine that during an epidemic, we need 1000 masks per day, *a*, are needed during an epidemic and in the warehouse of the public health agency. The variable of integration is time, *t*, in the number of days: Integrating *a* with respect to *t* gives 1000*t* masks, which equals the total public health resources needed over time.

Power rule of integration: When integrating the power of *x*, increase the exponent by one and divide by the new exponent.

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C \text{ for } n \neq -1$$

Sum rule of integration: The integral of a sum is the sum of the integrals.

$$\int (f(x) + g(x))\, dx = \int f(x)\, dx + \int g(x)\, dx$$

Difference Rule of Integration: The integral of a difference is the difference of the integrals.

$$\int (f(x) - g(x))\, dx = \int f(x)\, dx - \int g(x)\, dx$$

Integration by Substitution: This method is used for integrating composite functions.

Let $u = g(x)$. Then, integrate $f\ (u)$ with respect to u:

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du$$

These rules can be applied in various contexts, such as calculating the total number of disease cases over a period from a known rate of infection or understanding the total effect of a public health policy by integrating the rate of change of a health outcome. These integrations provide a broader view of public health dynamics, which is essential for designing and evaluating public health intervention strategies.

Problem 2: Using integration. The public health epidemiologist for this state determined that a flu epidemic is indeed present within its borders. Let P(t) be the number of persons sick with the flu at time t, where time is measured in days from the beginning of the epidemic and P(0) = 100. Suppose that after t days, the flu is spreading at a rate of $120t–3t^2$ people per day. Find the formula for P (t).

Solution:

Set up the integral: To find *P(t)*, we need to integrate the rate function $P'(t) = 120t–3t^2$:

$$P(t) = \int(120t–3t^2)\ dt$$

Integrate term by term: We use the power rule for integration to find *P(t)*.

$$P(t) = \int 120t\, dt - \int 3t^2\, dt$$

$$P(t) = 120 \cdot \frac{t^2}{2} - 3 \cdot \frac{t^3}{3} + C$$

$$P(t) = 60t^2 - t^3 + C$$

Determine the constant C using the initial condition: We know *P(0) = 100*.

$$P(0) = 60 \times 0^2 - 0^3 + C = 100$$
$$C = 100$$

Write the final formula: Incorporating *C* into *P(t)*.

$$P(t) = 60t^2 - t^3 + 100$$

Interpretation for public health epidemiology: The function $P(t) = 60t^2 - t^3 + 100$ represents the number of individuals sick with the flu at any given time *t* during the epidemic. This model is crucial for public health officials to have the ability to predict the disease's progression and strategize interventions for the state population. The problem demonstrates the power of integration to accumulate the rate of flu spread over time, offering insights into the epidemic's progression. It underscores the important role of integration in understanding the total impact of an outbreak, which is critical for public health resource allocation and long-term public health strategic planning.

FIRST-ORDER DIFFERENTIAL EQUATIONS

Differential Equation: The First-order Differential Equation

First-order differential equations, particularly modeling exponential growth and decay, are crucial for predicting disease spread and as-

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sessing intervention efficacy.[9,10] These models are often the basis of epidemiologic modeling activities. In practice, these models are useful for public health officials in developing public health strategies, allocating resources, and planning health service management during and after epidemics.[11-13]

Exponential functions, represented by equations such as $P(t) = 2^t$ for population growth exponential growth (where time is t=0,1,2,…,*n*) and base 2, represent a population size that doubles every unit of time. A differential equation is an equation involving an unknown function and one or more derivatives; a first-order differential equation contains an unknown function and only the first derivative. The function can model processes where the rate of change of a quantity is proportional to the quantity itself—a characteristic of many biological and epidemiological phenomena.

A differential equation in epidemiology for public health practice is a mathematical expression that describes how a disease spreads over time in an outbreak. It combines the rate of change with the current status of the outbreak to predict the future spread of the disease in a population. By understanding the dynamics of these rates of change in the differential equation, epidemiologists can better understand disease spread, and this knowledge could help with public health resource allocation and evaluating public health control measures.

In other words, first-order differential equations model dynamic changes, such as disease spread in a population or public health service utilization. These equations, which connect a variable's rate of change to the variable itself, offer insightful models that describe how these rates evolve over time. The solutions to differential equations in public health provide concrete functions or models that describe the behavior of epidemiological variables over time.[14]

For instance, differential equations can represent the rapid spread of an infection or the varying demand for health services within a community (Problem 3). Through the method of separation of variables, epidemiologists can derive explicit formulas, providing a predictive framework for understanding epidemiological trends and service usage patterns. These models are pivotal in formulating effective public health strategies, allowing public health professionals to anticipate and respond to changing health dynamics in the community, thereby enhancing intervention planning and resource allocation.

Problem 3: Using the First-Order Differential Equation. In a public health investigation, understanding the growth of a bacterial culture is crucial for predicting potential outbreaks and implementing control measures. The public health epidemiologist understands that this problem involves determining a function that models the growth of a bacteria culture over time, given that the growth rate is proportional to its current size. Let *P(t)* be the number of bacteria present at time *t*.

Given:

- At time $t = 0$, there are 20,000 bacteria.
- After 5 hours, the bacteria count is 400,000.
- The growth rate of the bacteria is proportional to its size.

Objective:

Determine a function *P(t)* that expresses the size of the bacteria culture as a function of time *t*, measured in hours.

Solution:

Step 1: Understanding the Model

Use an exponential growth model, which is appropriate here since the growth rate of the bacteria is proportional to its size. The general form of this model is:

$$P(t) = P_0 \times e^{kt}$$

where:

- P(t) is the population at time t,
- P_0 is the initial population size,
- e is the base of the natural logarithm,
- k is the constant of proportionality (growth rate),
- t is the time.

Step 2: Setting Up the Initial Condition

It is known that at t=0, P(0)=20,000. Plugging these values into the model gives us:

$$20{,}000 = P_0 \times e^{k.0}$$
$$20{,}000 = P_0 \times 1$$
$$P_0 = 20{,}000$$

Step 3: Using the Additional Information

At t=5 hours, P(5)=400,000. Use this to find the growth rate k:

$$400{,}000 = 20{,}000 \times e^{5k}$$
$$20=e^{5k}$$

Now, solve for k:

$$\ln(20) = \ln(e^{5k})$$
$$\ln(20) = 5k$$
$$k = \frac{ln(20)}{5} = 0.5991 \approx 0.60$$

Step 4: Formulating the Function

Now plug k back into the general model to get our specific function:

$$P(t) = 20{,}000 \times e^{0.5991 \times t}$$

Interpretation for Public Health Epidemiology

This function, P(t), models the size of the bacterial culture as a function of time t. The growth rate k is approximately 0.60. Now, we can write the function P(t) as:

$$P(t) = 20{,}000 \times e^{0.60\times t}$$

This function provides the number of bacteria present at any time t, allowing the epidemiologist to predict the growth of the bacteria culture over time, which is crucial for understanding and managing the potential public health implications.

This function helps us understand how quickly the bacterial population is growing, which is crucial for predicting and controlling outbreaks. Knowing the growth rate and how the population changes over time allows epidemiologists to work with other public health professionals to develop strategies to mitigate the spread, such as implementing sanitation measures or advising on antibiotic use. By analyzing the function, public health officials can predict when the bacteria culture might reach levels that pose a health risk, enabling proactive measures to prevent infections and protect public health.

PROBABILITY AND APPLIED CALCULUS

Probability Concepts and Applied Calculus Used in Statistical Techniques for Public Health Epidemiology

The interplay between probability and applied calculus forms the foundation of statistical techniques essential for epidemiology in public health practice. Probability and applied calculus complement each other, enhancing the analysis and interpretation of public health data and offering crucial insights into the design and execution of public health studies and interventions. These mathematical concepts enable epidemiologists to understand community public health service utilization, predict the outcome of public health interventions, and model communicable disease trends. Consider the following five topics:

The Central Limit Theorem: As commonly understood, this theorem asserts that the distribution of sample means approximates a normal distribution as the sample size becomes large, regardless of the population's distribution. For example, if we repeatedly sample the number of a particular disease case in different towns (different sample means), those means will form a normal distribution.

Example Formula: If $X_1, X_2, \ldots X_n$ are random samples from a population with mean μ and variance σ^2/n, the sample mean is given by

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

and has a distribution approaching normality as n increases, with mean μ and variance $\frac{\sigma 2}{n}$

Discrete and continuous distributions: Discrete distributions like the binomial distribution model count data, such as the number of new cases of a disease, while continuous distributions like the normal distribution apply to continuous data, such as patient recovery times.

Example formula (Binomial Distribution): The probability of observing x successes in n trials is given by

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where p is the probability of success on a single trial.

Example formula (Continuous Distribution: Normal Distribution): The probability density function (pdf) of a normal distribution is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Where:
- x is the public health outcome variable of interest,
- μ is the mean of the distribution,
- σ is the standard deviation of the distribution,

$$\sigma\sqrt{2\pi}$$

- is the normalization factor ensuring the total area under the curve is 1,
- e is the base of the natural logarithm.

Brief Explanation:

- The mean μ determines the center of the distribution, where the curve peaks.

- The standard deviation σ controls the spread of the distribution; larger values of σ result in a wider, flatter curve.

- The term

$$\frac{1}{\sigma\sqrt{2\pi}}$$

ensures the total probability (area under the curve) equals 1, a fundamental property of probability distributions.

- The exponential term $e^{-1/2\left(\frac{x-\mu}{\sigma}\right)2}$ provides the shape of the bell curve, causing the distribution to decrease symmetrically as x moves away from μ.

In epidemiology, the normal distribution is often used to model continuous data like recovery times in a population, where most recoveries cluster around a mean value with fewer instances of extremely short or long recovery times. Understanding this distribution aids in predicting public health outcomes and designing public health interventions based on the expected distribution of a measured variable.

Expected Value and Mean: The expected value in probability provides a measure for predicting outcomes, akin to the mean in statistics.

Example: In a study estimating the average number of days a patient stays in the hospital, if the probability of staying x days is $P(X=x)$, the expected number of days is

$$E(X) = \Sigma x \cdot P(X = x)$$

Law of large numbers: This law underlines the reliability of large sample sizes in approximating the population mean, which is essential for the validity of epidemiological studies. This concept has implications for epidemiology in public health practice:

• **Sample Size Considerations:** When planning epidemiological studies, ensuring a sufficiently large sample size is crucial. It enhances the reliability of study findings by reducing the impact of random fluctuations or anomalies in the data.

• **Estimating Population Parameters:** Epidemiologists depend on samples to estimate population parameters like disease prevalence or the average effect of an intervention. The law of large numbers provides the theoretical foundation for assuring that these estimates are accurate when the sample size is large.

• **Policy and Intervention Design:** Public health policies and interventions are often based on findings from epidemiological studies. The law of large numbers ensures that decisions made based on sufficiently large and representative sample data are likely to be effective in the broader population.

• **Understanding Randomness:** In public health data, randomness and variability are inherent. The law of large numbers helps epidemiologists distinguish between true signals and random noise, ensuring that interventions are based on reliable evidence.

Statistical Tools often used in public health epidemiology:

• **Linear Regression:** Used to model the relationship between a continuous dependent variable and one or more independent variables.

Example Formula:

$$Y = \beta_0 + \beta_0 X_1 + \varepsilon$$

where Y is the dependent variable, X_1 is the independent variable, β_0 is the intercept, β_1 is the slope, and ε is the error term.

• **Logistic Regression:** Suitable for binary outcome data, providing odds ratios that describe the relationship between independent variables and a binary outcome.

Example Formula:

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X_1$$

where p is the probability of the event occurring (Problem 4).

• **Time-to-Event Models:** One is the Kaplan-Meier estimator, which is used to estimate the survival function from lifetime data.

Example Formula:

$$S(t) = \prod_{t_i \le t} \frac{n_i - d_i}{n_i}$$

where $S(t)$ is the probability of survival until time t, n_i is the number of subjects at risk at time t_i, and d_i is the number of events (e.g., deaths) at time t_i.

These topics highlight the crucial role of probability and calculus in epidemiology, demonstrating how these mathematical concepts are applied in analyzing health data and informing public health decisions.

Problem 4: Using Logistic regression

Logistic regression is a statistical method that employs probability and calculus to analyze data where the outcome variable is categorical, often binary. In epidemiology for public health practice, logistic regression is commonly used to study the relationship between one or more predictor variables and a binary outcome (e.g., the presence or absence of a disease).

For instance, researchers might use logistic regression to determine the odds of developing a particular public health condition based on various risk factors, such as age, diet, or genetic predispositions. The logistic model is formulated, generally, as:

$$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X_1$$

where p is the probability of the event of interest (e.g., contracting a disease), $\beta_0, \beta_1, \ldots, \beta_n$ are the coefficients to be estimated, and $x_1, x_2, \ldots, x_n$ are the predictor variables.

This model leverages applied calculus, particularly in the estimation of the coefficients through maximum likelihood estimation, a process that involves finding the set of coefficients that maximize the likelihood function. The calculus concept of differentiation is employed to determine the maximum of this function.

By applying logistic regression using any software package of choice, epidemiologists can quantify how different factors influence the likelihood of a public health outcome, guiding public health strategies. For example, understanding which factors significantly increase the risk of a disease (often using the odds ratio) can inform targeted intervention programs, resource allocation, and health policy development, ultimately aiming to mitigate the disease's impact on the population. Logistic regression is a statistical technique supported by applied calculus to evaluate the risk factors associated with public health outcomes. This problem shows the potential role of applied calculus in crafting evidence-based policy-making and targeted health interventions.

Probability and applied calculus are vital for understanding disease risk, incidence, and spread in a population. Probability models like binomial, normal, and Poisson distributions assess public health event likelihoods, while applied calculus aids in modeling rates of change and cumulative quantities, essential for disease dynamics and intervention effects. These mathematical tools underpin statistical techniques for public health data analysis and model development, which are crucial for designing and evaluating public health interventions. Together, these methods provide a framework for informed public health decision-making and the effective development of public health strategies, showing their practical significance in addressing real-world health challenges.

CONCLUSION

Drawing on foundational texts,[5,6] this essay sought to introduce and underscore the relationship between applied calculus and public health epidemiology,[4] demonstrating how mathematical principles bolster analyses that underpin public health interventions. Derivatives provide insight into disease rates of change, informing targeted interventions, while integrals aggregate data, offering a comprehensive view of health event impacts. The use of first-order differential equations in disease spread modeling and statistical methods in data analysis underscores the pivotal role of calculus in epidemiological research and strategic public health decision-making. Applied calculus is a practical tool that equips epidemiologists engaged in public health practice with the ability to calculate rates of change, predict trends, devise effective strategies, and enhance public health outcomes, illustrating its vital contribution to advancing public health strategies and improving population health status.

FURTHER STUDY

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DISCLAIMER

The views expressed in this paper are those of the author and do not represent the official position of the US Government nor the US Department of Veterans Affairs.

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